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LETTER TO THE EDITOR

The electrical characteristics of random *RC* networks and the physical origin of 1/*f* noise

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Abstract
Simulations of the electrical noise in large networks of randomly positioned resistors and capacitors show a network noise power spectral density that varies as $1/f^{\alpha}(0 < \alpha < 1)$. This is found to occur across the frequency range over which the effective network dielectric loss, $\tan\delta$, is almost constant. It is suggested that the origin of 1/*f* noise, for some materials, is an inhomogenous microstructure that acts as an effectively random network of conductive and capacitive regions.

The physical origin of the 1/*f* noise that is widely observed in electronic devices remains a topic of intense research and speculation. Amongst the many proposed explanations of 1/*f* noise is the suggestion that it has the same physical origin as the frequency-independent loss tangent or constant phase angle (CPA) characteristic of many dielectric solids [1,2]. Recently, we have shown [3] that large electrical networks of randomly positioned resistors and capacitors exhibit CPA over many decades of frequency. This indicates that the appearance of CPA, or frequency-independent loss tangent, in the electrical characteristics of many materials may be explained by their microstructures being, effectively, a random network of conductive and capacitive regions. The possibility of such microstructural inhomogeneity is considerable in materials of many types, making this a plausible and widely applicable explanation of CPA. In this letter we review the conditions for 1/*f* noise in materials which exhibit a frequency-independent loss tangent. This is followed by the presentation of some simulation studies of the noise in large random *RC* networks which is found to be $1/f^{\alpha}$ $(0 < \alpha < 1)$ noise, rather than pure 1/*f* noise.

The connection between 1/*f* noise and frequency-independent loss tangent can be traced to Van der Ziel [1] and it has been considered at length quite recently by Klienpenning [2]. Following Klienpenning, the power spectral density, S_v, of the open-circuit noise voltage across the terminals of a leaky capacitor is given by:

$$S_v = 4\,kT\,Re(Z) = \frac{4\,kTR}{1+\omega^2R^2C^2} = \frac{kT\varepsilon'/\varepsilon''}{\omega C\left[1+(\varepsilon'/\varepsilon'')^2\right]}$$

$$S_v \approx \frac{4\,kT\tan\delta}{\omega C} \qquad \text{for} \qquad \varepsilon' \gg \varepsilon'' \tag{1}$$

where ε' and ε'' are the real and imaginary parts of relative permitivity and R and C are the equivalent shunt resistance and capacitance, $L/(\omega\varepsilon''\varepsilon_0 A)$ and $\varepsilon'\varepsilon_0 A/L$ respectively, of a sample in the form of a capacitor with plate area A and plate spacing L. The loss tangent, $\tan\delta = \varepsilon''/\varepsilon'$ and other symbols have their usual meanings. For 1/*f* noise, Klienpenning noted that $\tan\delta$ must be frequency independent. However, it is also necessary that the capacitance C is frequency independent. Jonscher [4,5] has collected a large number of examples of experimental data showing a frequency-independent loss tangent, or CPA, across many decades of frequency. This property is a component of what Jonscher proposed [6] to be the universal dielectric response (UDR) of solids. This response is dominated by power law frequency dependences of the permittivity components $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$, i.e.:

$$\begin{aligned} &\varepsilon'(\omega) \propto \omega^{-n} \\ &\varepsilon''(\omega) \propto \omega^{-n} \qquad 0 < n < 1 \qquad \text{and} \\ &\varepsilon''(\omega)/\varepsilon'(\omega) = \cot[(1-n)\pi/2] \end{aligned} \tag{2}$$

Hence, $\tan\delta = \varepsilon''(\omega)/\varepsilon'(\omega) = \cot[(1-n)\pi/2] = \text{constant}$.

In equation (1), $C \propto \varepsilon'(\omega) \propto \omega^{-n}$ and hence:

$$S_v \propto \frac{4\,kT\,\cot\,[(1-n)\pi/2]}{\omega^{1-n}} \tag{3}$$

This relationship shows that pure 1/*f* noise is not expected from a dielectric which has the UDR unless the power law exponent $n = 0$. This would be unusual as n values are found across the whole range 0 to 1. Instead, $1/f^{\alpha}$ noise is predicted, where $\alpha = 1 - n$.

An assumption in reaching the approximation in equation (1) is that $\varepsilon'(\omega) \gg \varepsilon''(\omega)$. This condition, however, is not essential for $1/f^{\alpha}$ noise. Provided $\tan\delta$ is constant, $S_v \propto 1/\omega C$ irrespective of the relative magnitudes of $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$.

Our random *RC* networks reproduce the above UDR characteristics. In these networks the exponent n is determined by the relative proportions of resistors and capacitors, i.e. n is the proportion of the network occupied by resistors. Hence, for a network to show pure 1/*f* noise it would be necessary for the resistor content to approach zero. A physical system in which this condition might be satisfied is a granular, essentially insulating, solid permeated by a grain boundary conducting phase.

The noise characteristics of the random *RC* networks can be assessed, independently of the above analysis, using the noise analysis option proved in the electrical network simulation software, SIMetrix [7]. This is the commercial software based on SPICE 3f.5 that was used in the earlier work [3]. Operated in the noise analysis mode, it assigns room temperature Johnson noise to each resistor in an *RC* network and computes the total network noise across a specified frequency range. This noise analysis has been applied to some of the large (512 component) 2D networks randomly filled with resistors and capacitors studied by us earlier [3].

The results shown in figure 1 were obtained from a network in which 60% of the components were 1 kΩ resistors and the remaining 40% were 1 nF capacitors. This was selected as being representative of networks with compositions above the percolation threshold (50%R 50%C) for conduction. The real and imaginary components of the effective network permittivity, shown in figure 1(a), both exhibit a power law decrease with frequency across the range ~1 kHz to ~1 MHz. The agreement is good with the expected [3] $\omega^{-0.6}$ power law dependence for a network in which 0.6 of the components are resistors. The frequency dependence of $\tan\delta$ is shown in figure 1(b). It is seen to be close to constant across the same frequency range, ~1 kHz to ~1 MHz, and in this region to have a value close to $\cot(0.4\pi/2)$, predicted by equation (2). The simulated electrical noise of the network is shown in figure 1(c). It exhibits a frequency dependence that is very close to the predicted, equation (3), $\propto \omega^{-0.4}$ behaviour across almost four decades of frequency. It was shown [3] that the frequency range covered by the power law behaviour of $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$ was determined by the size of the

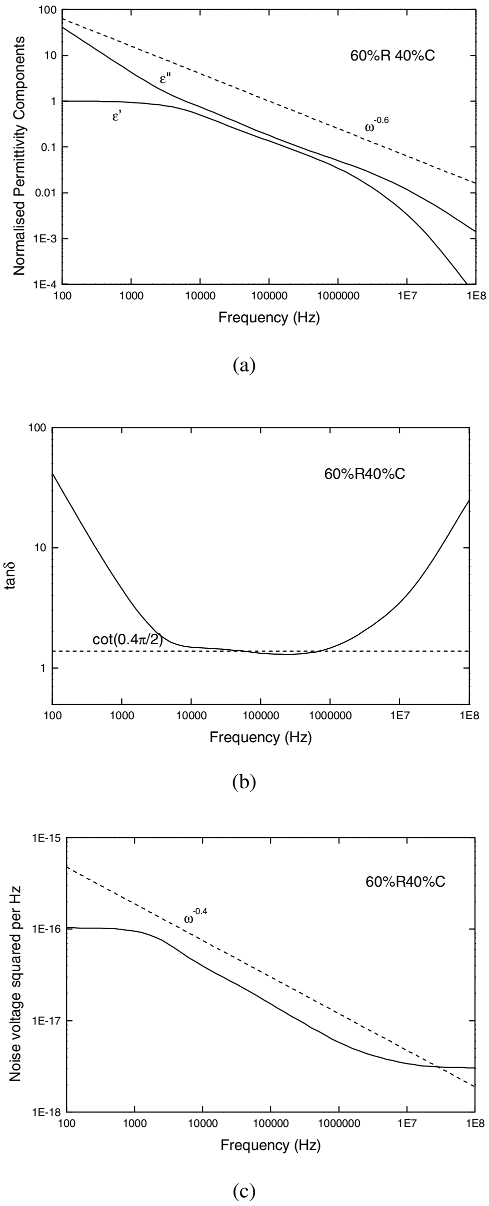
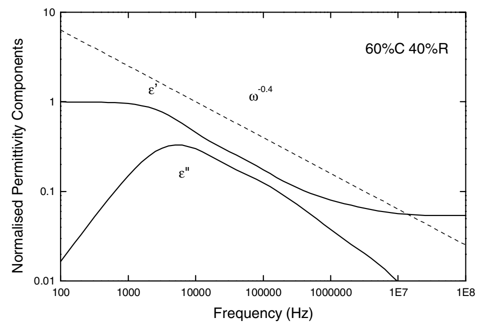


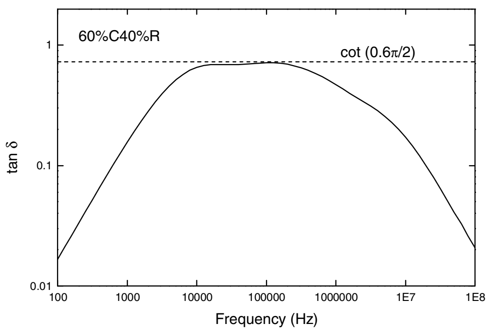
Figure 1. The frequency dependence of: (a) the effective permittivity and dielectric loss, (b) tan δ and (c) electrical noise obtained from simulations of a 2D square network randomly filled with 512 components of which 60% were 1 kΩ resistors and 40% were 1 nF capacitors.

network. A wider frequency range of adherence to the $\propto \omega^{-0.4}$ noise characteristic would be expected for significantly larger networks.

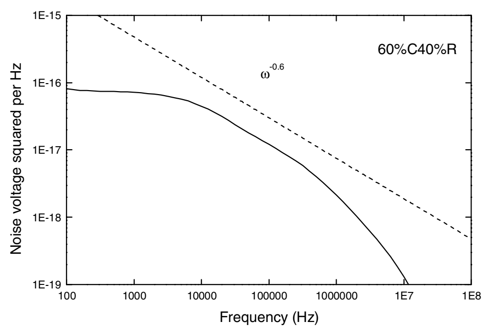
Equivalent results for a $60\%C$ $40\%R$, sub-percolation threshold dielectric network, are shown in figure 2. Although the constant tan δ region (figure 2(b)) was found to cover a narrower range of frequencies than for the conductive network (figure 1(b)) the predicted noise frequency dependence (equation (3)) is evident in figure 2(c). It is interesting to find



(a)



(b)



(c)

Figure 2. The frequency dependence of: (a) the effective permittivity and dielectric loss, (b) tan δ and (c) electrical noise obtained from simulations of a 2D square network randomly filled with 512 components of which 40% were 1 kΩ resistors and 60% were 1 nF capacitors.

the frequency dependence of the noise exceeding 1/*f* at highest frequencies. Again a wider $\propto \omega^{-0.6}$ range would be expected for a larger network. In addition, *R* and/or *C* being assigned a broad distribution of values, instead if the fixed values employed here, would significantly broaden the $\propto \omega^{-0.6}$ range.

The electrical characteristics of random *RC* networks have many features in common with the anomalous UDR found in a wide range of solids. These characteristics of random *RC* networks can be readily explained [3] by basic AC theory and reproduced by simulation. This has led us to suggest that solids which exhibit these characteristics are internally inhomogeneous with a microstructure that that is effectively a random network of dielectric regions and regions that are electrically conducting. The attractions of this suggestion are that such microstructures could occur in all classes of materials, accounting for the ubiquity of the anomalous electrical properties of solids and that these properties are a natural consequence of the microstructural network and do not necessitate any revision of our basic understanding of electrical behaviour at the atomic level. Dyre and Schroder [8] have recently reviewed the universal ac conductivity of disordered solids and discussed in some detail the possibility of it being explained by the existence in these solids of extensive microscopic electrical networks of parallel RC elements. In this letter it has been shown that $1/f^{\alpha}$ noise is a property of random RC networks, suggesting microstructural inhomogeneity to be a cause of $1/f$ noise.

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